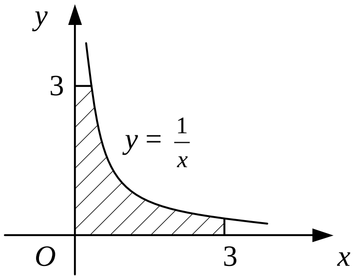
C4 INTEGRATION Worksheet P

1



The diagram shows the curve with equation $y = \frac{1}{x}$, $x > 0$.

The shaded region is bounded by the curve, the lines $x = 3$ and $y = 3$ and the coordinate axes.

a Show that the area of the shaded region is $1 + \ln 9$. **(5)**

b Find the volume of the solid generated when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. **(5)**

2 Given that

$$I = \int_0^4 x \sec\left(\tfrac{1}{3}x\right)\, dx,$$

a find estimates for the value of I to 3 significant figures using the trapezium rule with

i 2 strips,

ii 4 strips,

iii 8 strips. **(6)**

b Making your reasoning clear, suggest a value for I correct to 3 significant figures. **(2)**

3 The temperature in a room is 10°C. A heater is used to raise the temperature in the room to 25°C and then turned off. The amount by which the temperature in the room exceeds 10°C is θ°C, at time t minutes after the heater is turned off.

It is assumed that the rate at which θ decreases is proportional to θ.

a By forming and solving a suitable differential equation, show that

$$\theta = 15e^{-kt},$$

where k is a positive constant. **(6)**

Given that after half an hour the temperature in the room is 20°C,

b find the value of k. **(3)**

The heater is set to turn on again if the temperature in the room falls to 15°C.

c Find how long it takes before the heater is turned on. **(3)**

4 **a** Find the values of the constants p, q and r such that

$$\sin^4 x \equiv p + q \cos 2x + r \cos 4x.$$ **(4)**

b Hence, evaluate

$$\int_0^{\frac{\pi}{2}} \sin^4 x \, dx,$$

giving your answer in terms of π. **(4)**

5 **a** Find the general solution of the differential equation

$$\frac{dy}{dx} = xy^3. \qquad \textbf{(4)}$$

b Given also that $y = \frac{1}{2}$ when $x = 1$, find the particular solution of the differential equation, giving your answer in the form $y^2 = \text{f}(x)$. **(3)**

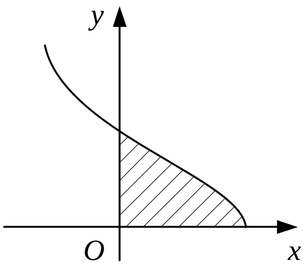
6 **a** Show that, using the substitution $x = e^u$,

$$\int \frac{2 + \ln x}{x^2}\, dx = \int (2 + u)e^{-u}\, du. \qquad \textbf{(3)}$$

b Hence, or otherwise, evaluate

$$\int_1^e \frac{2 + \ln x}{x^2}\, dx. \qquad \textbf{(6)}$$

7



The diagram shows the curve with parametric equations

$$x = \cos 2t, \quad y = \tan t, \quad 0 \le t < \frac{\pi}{2}.$$

The shaded region is bounded by the curve and the coordinate axes.

a Show that the area of the shaded region is given by

$$\int_0^{\frac{\pi}{4}} 4\sin^2 t\, dt. \qquad \textbf{(4)}$$

b Hence find the area of the shaded region, giving your answer in terms of π. **(4)**

c Write down expressions in terms of $\cos 2A$ for

i $\sin^2 A$,

ii $\cos^2 A$,

and hence find a cartesian equation for the curve in the form $y^2 = \text{f}(x)$. **(4)**

8

$$\text{f}(x) \equiv \frac{6 - 2x^2}{(x+1)^2(x+3)}.$$

a Find the values of the constants A, B and C such that

$$\text{f}(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{x+3}. \qquad \textbf{(4)}$$

The curve $y = \text{f}(x)$ crosses the y-axis at the point P.

b Show that the tangent to the curve at P has the equation

$$14x + 3y = 6. \qquad \textbf{(5)}$$

c Evaluate

$$\int_0^1 \text{f}(x)\, dx,$$

giving your answer in the form $a + b\ln 2 + c\ln 3$ where a, b and c are integers. **(5)**